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Change Point Estimation of Generalized Compound Rayleigh Distribution under Linex Loss Function

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Abstract

Consider A sequence of independent lifetimes x1, x2,....,xm,,xm+1,...xn (n≥3) were observed from Generalized Compound Rayleigh Distribution with parameter α,β,γ but it was found that there was a change in the system at some point of time m and it is reflected in the sequence after xm by change in sequences as well as change in the parameter values. The Bayes estimates of parameter γ and change point m are derived for asymmetric loss function known as Linex Loss Function under natural conjugate prior distribution. A simulation study is conducted, showing that the estimators perform well even when only of change is observed in the sequence.

Keywords: Change Point Estimation, Generalized Compound Rayleigh Distribution , Bayesian Method, Natural Conjugate Inverted Gamma Prior, Linex Loss Function.

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1.Introduction

Statistical decision theory deals with situations where decisions have to be made under a state of uncertainty, and its goal is to provide a rational frame work for dealing with such situations. The Bayesian approach is a particular way of formulating and dealing with statistical decision problems. More specifically, it offers a method of formulizing a priori beliefs and of combining them with the available observations, with the goal of allowing a rational derivation of optimal decision criteria. So in decision theory and estimation theory, a Bayes estimator is an estimator or decision rule that maximizes the posterior expected value of a utility function or minimizes the posterior expected value of a loss function also called posterior expected loss.

1.2 Loss Function

A symmetric loss function assumes that positive and negative error are equally serious. However, in some estimation problems such an assumption may be inappropriate. Cannfield (1970) points out that the use of symmetric loss function may be inappropriate in the estimation of reliability function. Over estimation of reliability function or average lifetime is usually much more serious than under estimation of reliability function or mean failure time.

Also an underestimate of the failure rate results in more serious consequences than an overestimation of the failure rate. This led to the statistician to think about asymmetrical loss function which have been proposed in statistical literature. Ferguson (1967), Zellner & Geisel (1968), Aitchision & Dunsmore (1975) and Berger (1980) have considered the linear asymmetric loss function. Varian (1975) introduced the following convex loss function known as LINEX. (Linear Exponential) Loss Function i.e. given as;

$$L(\Delta) = be^{a\Delta} - c\Delta - b\,; a, c \neq 0, b > 0 \qquad (1.2.1)$$

Where $\Delta = \hat{\theta} - \theta$. It is clear that L(0) = 0 and the minimum occurs when ab=c, therefore , $L(\Delta)$ can be written as

$$L(\Delta) = b[e^{a\Delta} - a\Delta - 1\,], a \neq 0, b > 0 \qquad (1.2.2)$$

Where a and b are the parameters of the loss function may be defined as shape and scale respectively. The loss function has been considered by Zellner (1986), Basu and Ebrahimi (1991) considered the $L(\Delta)$ as

$$L(\Delta) = b[e^{a\Delta} - a\Delta - 1\,],\ a \neq 0, b > 0 \qquad (1.2.3)$$

Where, $\quad \Delta = \frac{\hat{\theta}}{\theta} - 1$

andstudied the Bayesian estimates under this LlNEX loss function for exponential lifetime distribution. This loss function is suitable for the situation where overestimation of θ is more costly than its underestimation .This loss function L(Δ) havethe following properties:

(i) For a=1, the function is quite asymmetric about zero with overestimation being more costly than underestimation.

(ii) For $a < 0$, L(Δ)rises exponentially when $\Delta < 0$(underestimation) and almost linearly when $\Delta > 0$ (overestimation); and

(iii) For small values of |a| , L(Δ)is almost symmetric and not from a squared error losses function, Indeed, on expanding

$$e^{a\Delta} \simeq 1 + a\Delta + \frac{a^2\Delta^2}{2} \text{or} L(\Delta) \simeq \frac{a^2\Delta^2}{2}$$

is a squared error loss function . Thus for small values |a| , optimal estimates are not for different from those obtained with a squared error loss function.

1.3 Change or Shift Point

Physical systems manufacturing the items are often subject to random fluctuations. It may happen that at some point of time instability in the sequence of lifetimes is observed. Such observed point is known as Change or Shift point inference problem. Such Change or Shift point inference problem is useful in statistical quality control to study the Change or Shifting in process mean, Linear time series models, and models related to econometrics. The monographs, Broemeling and Tsurmi (1987) on structural changes and survey by Zack (1981) are useful references. Bayesian approach may play an important role in the study of such Change or Shift point problem and has been often proposed as a valid alternative in classical estimation procedure. A variety of Change or Shift point problems have studied in Bayesian frame work by many authors like Zellner (1986), Calabria and Pulcini(1994) and Jani and Pandya (1999).

1.4 Generalized Compound Rayleigh Distribution

The Generalized Compound Rayleigh Distribution is a special case of the three- parameter Burr type XII distribution. Mostert, Roux, and Bekker(1999) considered a gamma mixture of Rayleigh distribution and obtained the compound Rayleigh model with unimodal hazard function. This unimodal hazard function is generalized and a flexible parametric model is thus constructed, which embeds the compound Rayleigh model, by adding shape parameter. Bain and Engelhardt(1991) studied this distribution (also known as the Compound Weibull distribution (Dubey 1968) from a Poisson perspective. The p.d.f. of Generalized Compound Rayleigh model (GCRD).

$$f(x;\alpha,\beta,\gamma) = \alpha\gamma\beta^{\gamma}x^{\alpha-1}(\beta + x^{\alpha})^{-(\gamma+1)} \qquad x;\alpha,\beta,\gamma > 0 \qquad (1.4.1)$$

with Probability Distribution Function

$$\mathrm{F(x)} = 1 - (1 - \beta x^{\alpha})^{-\gamma} \qquad x;\alpha,\beta,\gamma > 0 \qquad (1.4.2)$$

Reliability function

$$\mathrm{R(t)} = \left(\frac{\beta}{\beta + t^{\alpha}}\right)^{\gamma} \qquad (1.4.3)$$

Hazard rate function

$$\mathrm{H(t)} = \alpha\gamma\frac{t^{\alpha-1}}{\beta + t^{\alpha}} \qquad (1.4.4)$$

The Generalized compound Rayleigh model includes various well-known p.d.f.'s, namely

(i) Beta-Prime p.d.f.(Patil, et al., 1984), if $\alpha = \beta = 1$

(ii) $\alpha = 1$

(iii) Burr XII p.d.f.(Burr,1942), if $\beta = 1$

Compound Rayleigh p.d.f.(Siddiqui &Weiss, 1963), if $\alpha = 2$

1.5 Bayesian Estimation of Change Point in Generalized Compound Rayleigh Distribution under Linex Loss Function (L.L.F.)

A sequence of independent lifetimes x_1, $x_{2,....}, x_{m,}, x_{m+1}, ... x_n$ $(n \geq 3)$ were observed from Generalized Compound Rayleigh Distribution with parameter α, β, γ but it was found that there was a change in the system at some point of time m and it is reflected in the sequence after x_m by change in sequences as well as change in the parameter values. The Bayes estimates of γ and m are derived for symmetric and asymmetric loss functions under natural conjugate prior distribution.

1.5.1 Likelihood, Prior, Posterior and Marginal

Let $x_1, x_2, \dots \dots, x_n,$ be a sequence of observed life times. First let observations $x_1, x_2, \dots \dots, x_n$ have come from Generalized Compound Rayleigh Distribution (G.C.R.D.) with probability density function as

$$f(x|\alpha, \beta, \gamma) = \alpha\, \beta^{\gamma} \gamma\, x^{(\alpha-1)} (\beta + x^{\alpha})^{-(\gamma+1)} \quad (x; \alpha, \beta, \gamma > 0) \tag{1.5.1.1}$$

Let 'm' is change point in the observation which breaks the distribution in two sequences as $(x_1, x_2, \dots \dots \dots .. x_m)$ & $\left(x_{(m+1),} x_{(m+2),} \dots \dots . x_n\right)$.

The probability density functions of the above sequences are

$$f_1(x) = \alpha_1 {\beta_1}^{\gamma_1} \gamma_1 x^{(\alpha_1-1)} (\beta_1 + x^{\alpha_1})^{-(\gamma_1+1)} \tag{1.5.1.2}$$

Where $x_1, \dots, x_m > 0; \alpha_{1,} \beta_1, \gamma_1 > 0$

$$f_2(x) = \alpha_2 {\beta_2}^{\gamma_2} \gamma_2 x^{(\alpha_2-1)} (\beta_2 + x^{\alpha_2})^{-(\gamma_2+1)} \tag{1.5.1.3}$$

Where $(x_{m+1}, \dots, x_n;\ \alpha_2, \beta_2, \gamma_2 > 0$

The likelihood functions of probability density function of the sequence are

$$L_1(x|\alpha_1, \beta_1, \gamma_1) = \prod_{j=1}^{m} f\left(x_j|\alpha_1, \beta_1, \gamma_1\right)$$

$$L_1(x|\alpha_1, \beta_1, \gamma_1) = (\alpha_1\gamma_1)^m U_1 e^{-\gamma_1 T_{1m}} \tag{1.5.1.4}$$

Where

$$U_1 = \prod_{j=1}^{m} \frac{x_j^{(\alpha_1-1)}}{\beta_1 + x_j^{\alpha_1}}$$

$$T_{1m} = \sum_{j=1}^{m} \log\left(1 + \frac{x_j^{\alpha_1}}{\beta_1}\right)$$

$$L_2(x|\alpha_2, \beta_2, \gamma_2) = \prod_{j=(m+1)}^{n} f\left(x_j \middle| \alpha_2, \beta_2, \gamma_2\right)$$

$$L_2(x|\alpha_2, \beta_2, \gamma_2) = (\alpha_2\gamma_2)^{(n-m)} U_2 e^{-\gamma_2(T_{1n}-T_{1m})} \quad (1.5.1.5)$$

where

$$U_2 = \prod_{j=m+1}^{n} \frac{x_j^{(\alpha_2-1)}}{(\beta_2 + x_j^{\alpha_2})}$$

and $\quad T_{1n} - T_{1m} = \sum_{j=(m+1)}^{n} \log\left(1 + \frac{x_j^{\alpha_2}}{\beta_2}\right)$

The joint likelihood function is given by

$$L\left(\gamma_1, \gamma_2 \middle| \underline{x}\right) \propto (\alpha_1\gamma_1)^m U_1\, e^{-\gamma_1 T_{1m}} (\alpha_2\gamma_2)^{n-m} U_2\, e^{-\gamma_2(T_{1n}-T_{1m})} \quad (1.5.1.6)$$

Suppose the marginal prior distribution of γ_1 and γ_2 are natural conjugate prior

$$\pi_1\left(\gamma_1, \underline{x}\right) = \frac{b_1^{a_1}}{\Gamma a_1} \gamma_1^{(a_1-1)} e^{-\gamma_1 b_1}; \qquad a_1, b_1 > 0, \gamma_1 > 0 \quad (1.5.1.7)$$

$$\pi_2\left(\gamma_2, \underline{x}\right) = \frac{b_2^{a_2}}{\Gamma a_2} \gamma_2^{(a_2-1)} e^{-\gamma_2 b_2}; \qquad a_2, b_2 > 0, \gamma_2 > 0 \quad (1.5.1.8)$$

The joint prior distribution of γ_1 ,γ_2 and change point 'm' is

$$\pi(\gamma_1, \gamma_2, m) \propto \frac{b_1^{a_1}}{\Gamma a_1} \frac{b_2^{a_2}}{\Gamma a_2} \gamma_1^{(a_1-1)} e^{-\gamma_1 b_1} \gamma_2^{(a_2-1)} e^{-\gamma_2 b_2} \quad (1.5.1.9)$$

where $\gamma_1, \gamma_2 > 0$ & $m = 1,2, \ldots \ldots \ldots (n-1)$

The joint posterior density of γ_1, γ_2 and m say $\rho\left(\gamma_1, \gamma_2, m/\underline{x}\right)$ is obtained by using equations (1.5.1.6) & (1.5.1.9)

$$\rho\left(\gamma_1, \gamma_2, m \middle| \underline{x}\right) = \frac{L\left(\gamma_1, \gamma_2/\underline{x}\right)\pi(\gamma_1, \gamma_2, m)}{\sum_m \iint_{\gamma_1,\gamma_2} L\left(\gamma_1, \gamma_2/\underline{x}\right)\pi(\gamma_1, \gamma_2, m)\, d\gamma_1 d\gamma_2} \quad (1.5.1.10)$$

$$\rho\left(\gamma_1, \gamma_2, m \middle| \underline{x}\right) = \frac{\gamma_1^{(m+a_1-1)}\, e^{-\gamma_1(T_{1m}+b_1)}\, \gamma_2^{(n-m+a_2-1)}\, e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}}{\sum_m \int_0^\infty \gamma_1^{(m+a_1-1)}\, e^{-\gamma_1(T_{1m}+b_1)}\, d\gamma_1 \int_0^\infty \gamma_2^{(n-m+a_2-1)}\, e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\, d\gamma_2}$$

Assuming

$\gamma_1(T_{1m} + b_1) = x$ & $\gamma_2(T_{1n} - T_{1m} + b_2) = y$

$\gamma_1 = \frac{x}{(T_{1m}+b_1)}$ & $\gamma_2 = \frac{y}{(T_{1n}-T_{1m}+b_2)}$

$d\gamma_1 = \frac{dx}{(T_{1m}+b_1)}$ & $d\gamma_2 = \frac{dy}{(T_{1n}-T_{1m}+b_2)}$

$$\rho(\gamma_1, \gamma_2, m/\underline{x}) = \frac{\gamma_1^{(m+a_1-1)}\ e^{-\gamma_1(T_{1m}+b_1)}\ \gamma_2^{(n-m+a_2-1)}\ e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}}{\xi(a_1,a_2,b_1,b_2,m,n)} \quad (1.5.1.11)$$

where

$$\xi(a_1, a_2, b_1, b_2, m, n) = \sum_{m=1}^{(n-1)} \frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}} \quad (1.5.1.12)$$

The Marginal posterior distribution of change point 'm' using the equations (1.5.1.6), (1.5.1.7) & (1.5.1.8)

$$\rho(m|\underline{x}) = \frac{L(\gamma_1, \gamma_2/\underline{x})\ \pi(\gamma_1)\ \pi(\gamma_2)}{\sum_m L(\gamma_1, \gamma_2/\underline{x})\ \pi(\gamma_1)\ \pi(\gamma_2)} \quad (1.5.1.13)$$

$$\rho(m|\underline{x}) = \frac{\int_0^\infty \gamma_1^{(m+a_1-1)}\ e^{-\gamma_1(T_{1m}+b_1)}\ d\gamma_1 \int_0^\infty \gamma_2^{(n-m+a_2-1)}\ e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\ d\gamma_2}{\sum_m \int_0^\infty \gamma_1^{(m+a_1-1)}\ e^{-\gamma_1(T_{1m}+b_1)}\ d\gamma_1 \int_0^\infty \gamma_2^{(n-m+a_2-1)}\ e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\ d\gamma_2}$$

Assuming

$\gamma_1(T_{1m} + b_1) = y$ & $\gamma_2(T_{1n} - T_{1m} + b_2) = z$

$\gamma_1 = \frac{y}{(T_{1m} + b_1)}$ & $\gamma_2 = \frac{z}{(T_{1n} - T_{1m} + b_2)}$

$d\gamma_1 = \frac{dy}{(T_{1m}+b_1)}$ & $d\gamma_2 = \frac{dz}{(T_{1n}-T_{1m}+b_2)}$

then

$$\rho(m|\underline{x}) = \frac{\frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)} \quad (1.5.1.14)$$

The marginal posterior distribution of γ_1 using equation (1.5.1.6) & (1.5.1.7) is given by

$$\rho(\gamma_1|\underline{x}) = \frac{L(\gamma_1, \gamma_2/\underline{x})\ \pi(\gamma_1)}{\int_0^\infty L(\gamma_1, \gamma_2/\underline{x}) \pi(\gamma_1) d\gamma_1} \quad (1.5.1.15)$$

$$\rho(\gamma_1|\underline{x}) = \frac{\sum_m \gamma_1^{(m+a_1-1)}\ e^{-\gamma_1(T_{1m}+b_1)} \int_0^\infty \gamma_2^{(n-m+a_2-1)}\ e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\ d\gamma_2}{\sum_m \int_0^\infty \gamma_1^{(m+a_1-1)}\ e^{-\gamma_1(T_{1m}+b_1)}\ d\gamma_1 \int_0^\infty \gamma_2^{(n-m+a_2-1)}\ e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\ d\gamma_2}$$

Assuming $\gamma_2(T_{1n} - T_{1m} + b_2) = y$, & $\gamma_2 = \frac{y}{(T_{1n} - t_{1m} + b_2)}$

then

$$\rho(\gamma_1|\underline{x}) = \frac{\sum_m e^{-\gamma_1(T_{1m}+b_1)}\, \gamma_1{}^{(m+a_1-1)}\, \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(\alpha_1,\alpha_2,\beta_1,\beta_2,m,n)} \quad (1.5.1.16)$$

The marginal posterior distribution of γ_2, using the equation (1.5.1.6) & (1.5.1.8) is given by

$$\rho(\gamma_2|\underline{x}) = \frac{\mathrm{L}(\gamma_1, \gamma_2/\, \underline{\mathrm{x}})\, \pi(\gamma_2)}{\int_0^\infty \mathrm{L}(\gamma_1, \gamma_2/\, \underline{\mathrm{x}})\, \pi(\gamma_2)\, d\gamma_2} \quad (1.5.1.17)$$

$$\rho(\gamma_2|\underline{x}) = \frac{\sum_m \int_0^\infty e^{-\gamma_1(T_{1m}+b_1)}\, \gamma_1{}^{(m+a_1-1)}\, [\gamma_2{}^{(n-m+a_2-1)}\, e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}]\, d\gamma_1}{\sum_m \int_0^\infty \gamma_1{}^{(m+a_1-1)}\, e^{-\gamma_1(T_{1m}+b_1)}\, d\gamma_1 \int_0^\infty \gamma_2{}^{(n-m+a_2-1)}\, e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\, d\gamma_2}$$

Assuming $\gamma_1(T_{1m} + b_1) = y$, $\gamma_1 = \frac{y}{(T_{1m}+b_1)}$

$$\rho(\gamma_2\, /\, \underline{x}) = \frac{\sum_m \left[\frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\right] \gamma_2{}^{(n-m+a_2-1)}\, e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}}{\xi(a_1,a_2,b_1,b_2,m,n)} \quad (1.5.1.18)$$

1.5.2 Bayes Estimators under Linex Loss Function (LLF)

The asymmetric loss function given Varian (1975), known as LLF, is defined as

$$L_2(\theta, d) = exp[k_1(d - \theta)] - k_1(d - \theta) - 1\, ;\; k_1 \neq 1, b = 1, \quad (1.5.2.1)$$

For the change or shift point 'm' the loss function is defined as

$$L_2(m, \widehat{m}_{BL}) = exp[k_1(\widehat{m}_{BL} - m)] - k_1(\widehat{m}_{BL} - m) - 1 \quad (1.5.2.2)$$

The Bayes estimate $\hat{\theta}_{BL}$ of θ under LLF is given by

$$\hat{\theta}_{BL} = -\frac{1}{k_1} \log E_\rho(-k_1\theta\,) \quad (1.5.2.3)$$

The Bayes estimate $\widehat{m}_{BL}$ of m under LLF equation (1.5.2.1) using marginal posterior of equation (1.5.1.14), is given as

$$\widehat{m}_{BL} = -\frac{1}{k_1} \log \left[\frac{\sum_m e^{-k_1 m}\, \frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right] \quad (1.5.2.4)$$

The Bayes estimate of $\hat{\gamma}_{1BL}$ of γ_1 using marginal posterior of equation (1.5.1.16) under LLF equation (1.5.2.1) is given by

$$\hat{\gamma}_{1BL} = -\frac{1}{k_1}\log E_\rho[\exp(-k_1\gamma_1)] \tag{1.5.2.5}$$

$$= -\frac{1}{k_1}\log\left[\frac{\sum_m e^{-k_1\gamma_1}\, e^{-\gamma_1(T_{1m}+b_1)}\,\gamma_1^{(m+a_1-1)}\,\frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]$$

$$\hat{\gamma}_{1BL} = -\frac{1}{k_1}\log\left[\frac{\sum_m \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}\int_0^\infty e^{-\gamma_1(T_{1m}+b_1+k_1)}\,\gamma_1^{(m+a_1-1)}\,d\gamma_1}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]$$

Assuming $\gamma_1(T_{1m}+b_1+k_1) = y$ & $\gamma_1 = \frac{y}{(T_{1m}+b_1+k_1)}$

$$\hat{\gamma}_{1BL} = -\frac{1}{k_1}\log\left[\frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{1m}+b_1+k_1)^{(m+a_1)}}\frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]$$

$$\hat{\gamma}_{1BL} = -\frac{1}{k_1}\log\left[\frac{\xi[a_1,(b_1+k_1),a_2,b_2,m,n]}{\xi(a_1,a_2,b_1,b_2,m,n)}\right] \tag{1.5.2.6}$$

The Bayes estimate of $\hat{\gamma}_{2BL}$ of γ_2 using marginal posterior of equation (1.5.1.18) under LLF equation (1.5.2.1) is given by

$$\hat{\gamma}_{2BL} = -\frac{1}{k_2}\log E_\rho[\exp(-k_2\gamma_2)] \tag{1.5.2.7}$$

$$\hat{\gamma}_{2BL} = -\frac{1}{k_2}\log\left[\frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\int_0^\infty \gamma_2^{(n-m+a_2-1)}\, e^{-\gamma_2(T_{1n}-T_{1m}+b_2+k_2)}\,d\gamma_2}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]$$

Assuming

$$\gamma_2(T_{1n}-T_{1m}+b_2+k_2) = y, \quad \gamma_2 = \frac{y}{T_{1n}-T_{1m}+b_2+k_2}$$

$$\hat{\gamma}_{2BL} = -\frac{1}{k_2}\log\left[\frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2+k_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]$$

$$\hat{\gamma}_{2BL} = -\frac{1}{k_2}\log\left[\frac{\xi[a_1,a_2,b_1,(b_2+k_2),m,n]}{\xi(a_1,a_2,b_1,b_2,m,n)}\right] \tag{1.5.2.8}$$

Numerical Comparison for Generalized Compound Rayleigh Sequences

As in chapter 2, We have generated 20 random observations from Generalized Compound Rayleigh distribution with parameter $\alpha = 2, \beta = 0.5$ and $\gamma = 2$. The observed data mean is 0.9639 and variance is 2.3071. Let the change in sequence is at 11^{th} observation, so the means and variances of both sequences $(x_1, x_2, \ldots, x_m)$ and $(x_{(m+1)}, x_{(m+2)}, \ldots, x_n)$ are $\gamma_1 = 1.2682$, $\gamma_2 = 0.5920$ and $\sigma_1{}^2 = 4.0395$ and $\sigma_2{}^2 = 0.1470$. If the target value of γ_1 is unknown, its estimating $(\hat{\gamma}_1)$ is given by the mean of first m sample observation given m=11, $\gamma = 1.268$.

Sensitivity Analysis of Bayes Estimates

In this section we have studied the sensitivity of the Bayes estimates with respect to changes in the parameters of prior distribution a_1, b_1, a_2 and b_2. The means and variances of the prior distribution are used as prior information in computing these parameters. Then with these parameter values we have computed the Bayes estimates of m, γ_1 and γ_2 under LINEX loss function (LLF) considering different set of values of (a_1, b_1) and (a_2, b_2).We have also considered the different sample sizes n=10(10)30 and parameter of loss function $\alpha = 2$. The Bayes estimates of the change point 'm' and the parameters γ_1 and γ_2 are given in table-3.1 under LLF. Their respective mean squared errors (M.S.E's) are calculated by repeating this process 1000 times and presented in same table in small parenthesis under the estimated values of parameters. All these values appear to be robust with respect to correct choice of prior parameter values and appropriate sample size. From the below table we conclude that -

The Bayes estimates of the parameters γ_1 and γ_2 of GCRD obtained with LLF are seems to be efficient as the numerical values of their mse's are very small for $\hat{\gamma}_{1BL}$ and $\hat{\gamma}_{2BL}$ in comparison with $\hat{m}_{BL}$.The Bayes estimates of the parameters are robust with correct choice of prior parameters as *a_1 =(1.5-2.0),a_2=(1.70-2.10), b_1=(1.75-2.25)and b_2=(1.80-2.20)* and all sample size.

Table 1.1. Bayes Estimates of m, γ_1& $\gamma_2$for GCRD sequences and their respective M.S.E.'s Under LLF

(a_1, b_1)	(a_2, b_2)	N	$\hat{m}_{BL}$	$\hat{\gamma}_{1BL}$	$\hat{\gamma}_{2BL}$
(1.25,1.50)	(1.50,1.60)	10	8.0203 **(32.8662)**	1.5811 **(0.7244)**	1.3026 **(7.9322)**
		20	17.3887 **(238.5909)**	1.4787 **(1.3644)**	2.8157 **(1.0854)**
		30	27.0848 **(628.7230)**	1.7528 **(0.9963)**	2.9818 **(0.4878)**
(1.50,1.75)	(1.70,1.80)	10	7.8454 **(32.0677)**	0.8360 **(2.0462)**	3.2129 **(0.1224)**
		20	17.4552 **(234.6840)**	1.1681 **(1.3419)**	1.0558 **(0.4866)**
		30	26.9288 **(626.2124)**	1.5740 **(0.9524)**	1.1744 **(0.1069)**
(1.75,2.0)	(1.90,2.0)	10	8.1574 **(34.1522)**	1.3374 **(0.2964)**	0.6929 **(1.0233)**
		20	17.5298 **(232.9754)**	2.6085 **(3.0442)**	0.8691 **(4.2477)**
		30	26.6271 **(641.9755)**	4.1469 **(0.0080)**	1.0669 **(1.4998)**
(2.0,2.25)	(2.10,2.20)	10	8.0208 **(35.6131)**	1.3007 **(0.0032)**	0.7501 **(2.6191)**
		20	17.6219 **(241.6463)**	1.1375 **(0.0001)**	1.0741 **(3.2409)**
		30	27.4464 **(653.2450)**	1.3938 **(1.2524)**	0.8447 **(0.5457)**
(2.25,2.50)	(2.30,2.40)	10	7.7056 **(34.5537)**	1.8255 **(0.0501)**	4.6209 **(0.5418)**
		20	17.6725 **(243.6634)**	1.1503 **(1.1562)**	0.7077 **(1.0411)**
		30	27.3153	1.3854	0.9043

			(645.2723)	**(.7058)**	**(0.7025)**
(2.50,2.75)	(2.50,2.60)	10	7.8521 **(34.7852)**	1.6976 **(0.3699)**	1.4339 **(1.1569)**
		20	18.0672 **(236.4615)**	1.7870 **(.2643)**	0.9677 **(1.4625)**
		30	27.1005 **(632.4395)**	1.0941 **(0.4872)**	1.0782 **(0.1066)**

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